

# New Method of Enhancing Lepton Number Nonconservation

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The lepton number nonconserving (LENNON) conversion of type  $e^- \rightarrow e^+$  in heavy atoms, when irradiated by intense laser beam, is considered to determine the Majorana nature and precise values of neutrino masses. When the photon energy is fine tuned, the LENNON process is greatly enhanced by both the resonance effect and a large occupancy of photons. The signal of this process would be a positron of definite energy, and a further nuclear  $\gamma$ -ray in a favorable case of transition to an excited nuclear level. By constructing a united target-detector system, it is possible to explore  $O[1\text{meV}]$  range of the neutrino mass parameter with a good positron energy resolution.

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Recent observations indicate that neutrinos have finite masses. The immediate question that arises is whether these masses are of Dirac or Majorana type. If they are of the Majorana type, the lepton number is not conserved. If this nonconservation were discovered, one should expect great impacts on other areas of physics, including the possible explanation of baryon-antibaryon imbalance in our universe [1]. The most extensively examined tool to study the Majorana nature of the neutrino mass is the neutrinoless double beta decay, and many proposals and experiments are already on-going [2].

We wish to propose alternatives to the neutrinoless double beta decay in order to investigate the nature of neutrino masses, both because the important issue of the lepton number nonconservation (abbreviated as LENNON) requires an independent experimental check, and because one should examine it in many other processes. We shall focus in the present work on photon stimulated electron conversion of type  $e^- \rightarrow e^+$  in heavy atoms, with no accompanying neutrino. The experimental signature of this process is a monochromatic positron of few MeV, and furthermore nuclear  $\gamma$ -ray in a favorable case of transition to nuclear excited states. Although simultaneous emission of low energy photons occurs, it might be difficult to detect them, because their energies are either down shifted or the same laser photon energy in the case of united target-detector system later discussed, and they might easily be confused amidst the background of high intensity beam. We give rates both for this process and the background of similar process of two accompanying neutrinos.

The basic low energy effective operator [3] that may arise in physics beyond the standard model is  $G_F^2 \tilde{m}_\nu l l \bar{q} q \bar{q} q$  where  $l$  is the lepton doublet and  $q$  is the quark doublet having the quantum number of the standard model. This class of effective interaction violates the lepton number by two units,  $\Delta L = 2$ , a class of simplest models predicting the amplitude in direct proportion to some combination of Majorana type of neutrino masses,  $\tilde{m}_\nu$ . Thus, the rate of the non-radiative LENNON con-

version of the type  $e^-(1s) \rightarrow e^+$ , even for a favorable case of  $^{112}\text{Sn}$ , becomes of order  $10^{-29}\text{year}^{-1}$  for the Majorana neutrino mass of order 0.1eV.

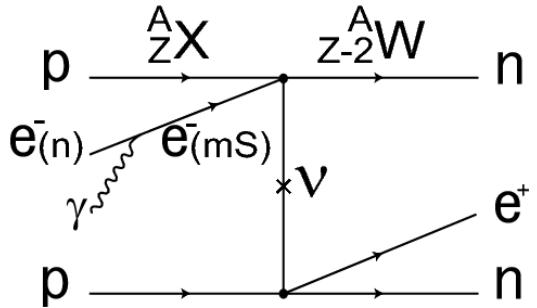


FIG. 1: Feynman diagram for photon irradiated LENNON process of atomic transition  $n \rightarrow ms$ .

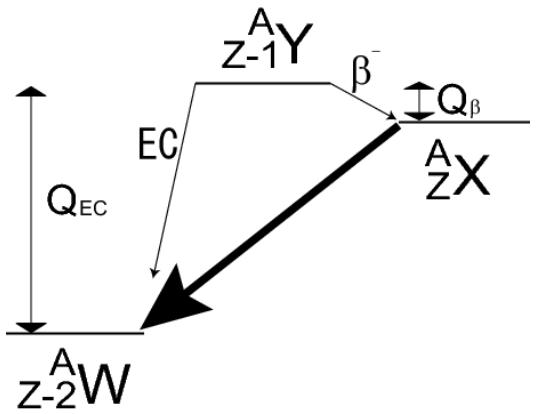


FIG. 2: Level structure for three adjacent nuclei of mass number  $A$  and atomic number  $Z$ .

We instead consider LENNON conversion stimulated by radiative absorption, of the type depicted in Fig. 1. Here  $^{A_Z}X$  is a nucleus of mass number  $A$  and atomic number  $Z$ . An electron occupying an atomic  $n$  state is upshifted by photon irradiation to an unoccupied  $ms$  level,

and later captured by nucleus, producing a positron by  $e^- \rightarrow e^+$  conversion. Another possibility is the positron emission first by weak interaction and a subsequent absorption of photon by atomic electron of  ${}^A_{Z-1}Y$ . The idea is to enhance the  $e^- \rightarrow e^+$  conversion with the help of both the resonance effect and a high occupancy of laser photons in a quantum level. Without irradiation of the initial photon, this is a process conjugate to the neutrinoless double beta decay, and can explore the same combination of the neutrino mass parameter  $m_{ee}$ , where  $m_{\alpha\beta} = \sum_k U_{\alpha k} U_{\beta k} m_k$  with  $m_k$  3 neutrino mass eigenvalues.

In Fig.2 we depict relevant nuclear levels for three adjacent nuclei of the same mass number. The final nucleus  ${}^A_{Z-2}W$  can be in an excited nuclear level, for which triple coincidence for detection including nuclear  $\gamma$ -ray may become possible. If both  $\beta^-$  decay and electron capture rates of the intermediate  ${}^A_{Z-1}Y$  nucleus are experimentally known, one may reliably estimate the rate for the radiative  $e^- \rightarrow e^+$  conversion. The best candidate with this regard is the isotope  ${}^{112}_{50}Sn$  [4].

A straightforward computation for the atomic  $n \rightarrow m$  transition followed by the capture gives the following matrix element;

$$\frac{eG_F^2 m_{ee}}{4\pi} 2\pi\delta(E_\gamma + \Delta E_{i,f} - E_+ + \Delta\epsilon_{nm}) \times \int d^3x d^3y \langle f | \frac{J(\vec{x}) \cdot J(\vec{y})}{|\vec{x} - \vec{y}|} | i \rangle \langle \vec{p}_+ | \overline{e^c}(\vec{x}) (1 - \gamma_5) e(\vec{y}) \sum_m \left( |m\rangle \frac{\langle m | H_\gamma | n \rangle}{E_\gamma - \Delta\epsilon_{nm} + i\tilde{\Gamma}_n/2} \right), \quad (1)$$

where  $e(\vec{y})$  etc. is the electron field operator. Here  $\Delta E_{i,f} = E_i - E_f$ , with energies  $E_{i,f}$  referring to nuclear levels of initial and final states. Thus,  $E_+ = \Delta E_{i,f} + E_\gamma + \Delta\epsilon_{nm}$  is the monochromatic positron energy. The important natural width  $\tilde{\Gamma}_n$  in (1) refers to that of the hole of  $n$  atomic state. The energy difference  $\Delta\epsilon_{nm} = \epsilon_n - \epsilon_m$  where  $\epsilon_{n,m}$  are energies of atomic levels.

The neutrino propagator has been replaced by the instantaneous Coulomb potential, which is allowed since for low energy electrons the energy transfer is small;  $|q_0| \ll |\vec{q}|$ . At low energies the electron wave functions can be taken out from the integral (1), and one may separate out the nuclear matrix element. Furthermore, by introducing an average inter-proton distance,  $R_n$ , one may take out the factor  $\langle 1/|\vec{x} - \vec{y}| \rangle = 1/R_n$  outside the nuclear matrix element. We use  $R_n \approx (0.82A^{1/3} + 0.58)fm$ , following [4].

The radiative electron conversion via  $ms$  atomic state has a cross section of the form,

$$\Gamma_{0\nu}^{(ms)} \frac{|\langle mS | H_\gamma | n, \gamma \rangle|^2}{(E_\gamma - \Delta\epsilon_{nm})^2 + \tilde{\Gamma}_n^2/4}, \quad (2)$$

where  $H_\gamma$  is QED interaction. Let us first consider the non-radiative conversion rate  $\Gamma_{0\nu}^{(ms)} = |\psi_{ms}(0)|^2 \sigma_{0\nu}$ . This product is an effective luminosity  $|\psi_{ms}(0)|^2$  of confined  $ms$  electron times the cross section for  $e^-$  (free) +  ${}^A_Z X \rightarrow e^+ + {}^A_{Z-2}W$  given by

$$\sigma_{0\nu} = \frac{G_F^4 \tilde{m}_\pi^2 |m_{ee}|^2}{16\pi^3} p_+ E_+,$$

$$\tilde{m}_\pi \equiv \int d^3x d^3y \frac{\langle f | J(\vec{x}) \cdot J(\vec{y}) | i \rangle}{|\vec{x} - \vec{y}|}. \quad (3)$$

Typically,  $\sigma_{0\nu} = O[10^{-66}cm^{-2}] |m_{ee}|^2 (p_+ E_+ / MeV^2)$ . When a photon is irradiated, the product is further multiplied by the Breit-Wigner function, which can be very large near the resonance,  $E_\gamma = \Delta\epsilon_{nm} \equiv E_0$ . In other words, the cross section ratio of the photon-irradiated conversion, (2) relative to the elementary  $\sigma_{0\nu}$ , is

$$K(E) = \frac{|\langle mS | H_\gamma | n, \gamma \rangle|^2 |\psi_{ms}(0)|^2}{(E - E_0)^2 + \tilde{\Gamma}_n^2/4} \approx \frac{2\pi^2 \Gamma_{ms \rightarrow n} |\psi_{ms}(0)|^2}{E_0^2 [(E - E_0)^2 + \tilde{\Gamma}_n^2/4]}. \quad (4)$$

Here the branching fraction  $B_{ms \rightarrow n} = \Gamma_{ms \rightarrow n} / \tilde{\Gamma}_n$  is of order unity.

The actual rate, when the photon beam of luminosity density  $I(E)$ , within unit energy bin and per unit area times unit time, is irradiated, is given by

$$R = \int dE I(E) \sigma_{\gamma e^- \rightarrow e^+}$$

$$= S \int dE \frac{I(E)}{(E - E_0)^2 + \tilde{\Gamma}_n^2/4}. \quad (5)$$

Note the dimensionless strength factor given by

$$S = 2\pi^2 \sigma_{0\nu} |\psi_{ms}(0)|^2 \frac{B_{ms \rightarrow n} \tilde{\Gamma}_n}{E_0^2}$$

$$\approx 2\pi \sigma_{0\nu} (Z\alpha)^6 m_e^3 \frac{B_{ms \rightarrow n} \tilde{\Gamma}_n}{E_0^2}. \quad (6)$$

The integral in (5) may readily be evaluated when the photon flux has little variation over the natural width  $\tilde{\Gamma}_n$

around  $E_0$ :

$$\int dE \frac{I(E)}{(E - E_0)^2 + \tilde{\Gamma}_n^2/4} \approx 2\pi \frac{I(E_0)}{\tilde{\Gamma}_n}.$$

When the photon beam is tuned and the resonance  $E_0$  is not missed, one further has  $I(E_0) \approx F(E_0)/\Delta E$ , where  $\Delta E$  is the energy resolution of photon beam and  $F(E_0) = \int dE I(E)$  is the total flux integrated over the region around  $E_0$ . This leads to the rate formula,

$$R \approx \frac{2\pi^2 F(E_0)}{E_0^2 \Delta E} B_{ms \rightarrow n} 2\pi |\psi_{ms}(0)|^2 \sigma_{0\nu}. \quad (7)$$

Each factor has a clear meaning; the first  $F(E_0)/(E_0^2 \Delta E (2\pi^2)^{-1})$  expressing the number of occupied photons within the relevant quantum phase space, the second  $B_{ms \rightarrow n}$  the branching fraction of order unity, the last  $|\psi_{ms}(0)|^2 \sigma_{0\nu}$  the  $e^- \rightarrow e^+$  conversion rate from  $ms$  state. Thus, the enhancement or reduction factor by photon irradiation relative to the capture rate  $|\psi_{1s}(0)|^2 \sigma_{0\nu}$  is given by  $4\pi^3 F(E_0) (|\psi_{ms}(0)|^2 / E_0^3) (E_0 / \Delta E) B_{ms \rightarrow n} / m^6$ . It is useful to define a quality factor  $Q$  of photon beam defined by  $Q = 4\pi^3 F(E_0) / (E_0^2 \Delta E)$ , which is numerically  $2 \times 10^{10} (F(E_0) / 1kW mm^{-2}) (E_0 / eV)^{-4} (10^{-9} E_0 / \Delta E)$ . An advantage of a strong laser beam, which can give a large  $Q$ , is obvious, when compared to X-ray which typically gives  $Q \ll 1$  (except contemplated X-ray laser).

One might wonder the validity of the linear rise with the laser power of the rate, eq.(7), because irradiated atoms may become transparent once electrons in the lower energy level are completely lifted to the higher energy level. However, we are considering the situation of equilibrated atoms between the two levels by constant irradiation of laser beam. In this case the rate formula (7) is still valid, with a minor multiplication of the population factor in the lower level. The formula valid in the large power limit is different and shall be presented elsewhere by one of the present authors (MY).

We shall next discuss how to estimate the rate  $\Gamma_{0\nu}$  for the non-radiative electron conversion by nucleus that appears in the fundamental formula (2). By truncating the nuclear level sum in  $\sum_k \langle f | J | k \rangle \langle k | J | i \rangle$  by a single ground state  $|k\rangle = |_{Z-1}^A Y, 1^+\rangle$ , one may replace the nuclear matrix element  $\langle k | J | i \rangle$  by the beta decay rate, and  $\langle f | J | k \rangle$  by the electron capture rate of nucleus  $|_{Z-1}^A Y$ . In the case of the nonradiative decay,  $e^- (1s) \rightarrow e^+$  process, the rate becomes

$$\Gamma_{0\nu} = \frac{3\pi^3 |m_{ee}|^2}{4R_n^2} \frac{p_+ E_+}{p_\nu^2 \Delta_\beta^5 I} \Gamma_{EC} \Gamma_\beta, \quad (8)$$

where  $\Gamma_{EC}, \Gamma_\beta$  are the electron capture and the beta decay rate of  $|_{Z-1}^A Y$ , and  $p_i$  are respective lepton momenta. The maximum  $\beta$  energy  $\Delta_\beta = Q_\beta + m_e - \epsilon_{1s}$ , and  $p_\nu = Q_{EC} - \epsilon_{1s}$ . Here  $I$  is a dimensionless phase space factor for the beta decay and  $1/30$  in the limit of

TABLE I: Useful characteristics for a few isotopes

Atom	abnd.%	$Q_{e^+}$ (keV)	$\tau_{0\nu \frac{1}{2} \text{yr}}   \frac{m_{ee}}{0.1 \text{eV}}  ^{-2}$	$\frac{\Gamma_{0\nu}}{\Gamma_{2\nu}(\delta E)}^a$
$^{78}_{36}\text{Kr}$	0.35	1846	$1.5 \times 10^{29}^b$	$1.6 \times 10^3$
$^{112}_{50}\text{Sn}$	0.97	901	$5.5 \times 10^{28}$	$1.8 \times 10^3$
$^{130}_{56}\text{Ba}$	0.11	1588	$2.3 \times 10^{29}$	$1.7 \times 10^3$

<sup>a</sup>We assume 0.1eV of the neutrino mass, and  $\delta E = 100 \text{keV}$ .

<sup>b</sup>For  $\text{Kr}$  half-life, we follow the calculation of [4] instead of eq.(8).

zero electron mass. We ignored the difference of  $1s$  electron wave function of  ${}^A_Z X$  and  ${}^A_{Z-2} W$ , whose error should be small, of order  $4/Z$ .

For instance, in the case of  $0^+ \rightarrow 0^+$  nuclear transition of  ${}^{112}_{50}\text{Sn} \rightarrow {}^{112}_{48}\text{Cd}$ , both intermediate  ${}^{112}_{49}\text{In}$  (of  $1^+$ )  $\beta^-$  decay and electron capture rates are known [5]. Thus, the rate computed according to eq.(8) is  $\approx (2 \times 10^{29} y)^{-1} (|m_{ee}| / 0.1 \text{eV})^{-2}$  for  ${}^{112}_{50}\text{Sn}$ . It appears that nuclear matrix elements are large, as pointed out in [4].

We numerically give the laser irradiated  $e^- \rightarrow e^+$  conversion rate for one nucleus target;

$$R \approx 5 \times 10^{-35} y^{-1} \times \frac{\Gamma_{0\nu}}{10^{-29} y^{-1}} \left( \frac{4}{m} \right)^6 \frac{E_\gamma}{\Delta E} \frac{F}{W mm^{-2}} \left( \frac{E_\gamma}{eV} \right)^{-4}. \quad (9)$$

The crucial factor to obtain a large enhancement of the rate is the inverse of resolution, and with  $\Delta E/E_\gamma = 10^{-9}$  available commercially, the rate is enhanced by  $\approx 5 \times 10^6 (4/m)^6$ , using a laser beam power  $1kW/mm^2$ . Linear increase with the intensity  $F$  of this event rate is the key check point of experimental verification of the process. A few isotopes which may give rates of  $\Gamma_{0\nu} > 10^{-31} y^{-1}$  are illustrated in Table 1.

The required resonance tuning might be a great practical obstacle since laser frequencies are superposition of quantized level differences. (Alternatively, the use of laser with a continuous spectrum might help greatly.) It is however possible to avoid this problem by selecting the same lasing medium as the target nucleus, thus we arrive at the concept of a united target-detector system. A good target must then be both lasing and an efficient  $e^- \rightarrow e^+$  converter. Candidates of such targets are limited, but the element  $\text{Kr}$  is a good example. The  $\text{Kr}^+$  laser uses the inverted population of  $4p$  atomic state, which falls down to  $4s$  by stimulated emission caused by irradiated laser [6]. In this basic process  $4s$  electron may very rarely be captured by  $0^+$   $\text{Kr}$  nucleus to emit a positron of energy  $\approx 1.8 \text{MeV}$ .

The isotope  ${}^{78}_{36}\text{Kr}$  after the  $e^- \rightarrow e^+$  conversion ends up with  ${}^{78}_{34}\text{Se}$ , which has a few excited energy levels of  $0^+$  below the  ${}^{78}_{36}\text{Kr}$  ground level. Thus, nuclear gamma rays are expected to give an opportunity of coincident measurement.

Let us estimate what might occur within an ideal  $Kr^+$  or gaseous laser device containing  $Kr$  such as  $KrF$  excimer laser. Suppose that the gas chamber of the device contains  $10^4 \text{ cm}^3$  of 1 ATM enriched  $^{78}Kr$ , which has  $\approx 3 \times 10^{23} \text{ }^{78}Kr$  atoms. The formula (9) tells that  $e^- \rightarrow e^+$  conversion occurs with a rate,  $\approx 10^4/y (I/\text{MW mm}^{-2})$ , assuming a resolution  $\Delta E/E_\gamma = 10^{-9}$  and  $\Gamma_{0\nu} = 10^{-29}/y$ . For the  $Kr^+$  laser,  $m = 4$ ,  $E_\gamma = 1.9\text{eV}$  for one of the main lines. The rate scales with the neutrino mass parameter as  $\Gamma_{0\nu} = 0.9 \times 10^{-29} y^{-1} |m_{ee}/0.1\text{eV}|^2$ , using nuclear matrix elements of [4] adopted to  $^{78}Kr$ , thus one may be able to explore  $|m_{ee}|$  down to  $1\text{meV}$ .

The important physics background to the present process is the corresponding process of two accompanying neutrinos caused by the second order weak interaction, which itself is of interest. We may estimate this rate by using the same approximation of one-level truncation. The result is given by the ratio of two processes,  $\Gamma_{0\nu}/\Gamma_{2\nu} \approx (20160\pi^2|m_{ee}|^2 m_\pi^2)/(\Delta^6 R_n^2)$  which is  $\approx 0.66 \times 10^{-4} |m_{ee}/1\text{meV}|^2 (m_\pi R_n)^{-2} (\Delta/\text{MeV})^{-6}$ .

On the other hand, the ratio when the positron energy is limited near the end point of  $e^+$  energy width  $\delta E$  is  $\Gamma_{0\nu}/\Gamma_{2\nu}(\delta E) \approx 45\pi^2 m_\pi^2 |m_{ee}|^2 / (R_n^2 (\delta E)^6)$ . With  $\delta E \approx 100\text{keV}$  and for  $^{78}Kr$ ,

$$\frac{\Gamma_{0\nu}}{\Gamma_{2\nu}(\delta E)} \approx O[18] \left| \frac{m_{ee}}{0.01\text{eV}} \right|^2 \left( \frac{\delta E}{100\text{keV}} \right)^{-6}. \quad (10)$$

The positron energy spectrum for two neutrino process is given by

$$d\Gamma_{2\nu} \propto E \sqrt{E^2 - m_e^2} (E - \Delta)^5 dE. \quad (11)$$

The signal of LENNON would be an excess of positrons near the end point of two neutrino processes. In the favorable case to decay into a nuclear excited level the background ( $2\nu$ ) process is suppressed by the phase space factor  $(E - \Delta)^5$  if the excited level has a relatively high energy.

In summary, it appears possible to explore the Majorana neutrino mass range down to  $1\text{meV}$  by laser irradiated  $e^- \rightarrow e^+$  conversion, when a lasing gas medium  $> 10^4 \text{ cm}^3$  of high power  $> 1\text{MW}$  or more, is used, along with a good measured positron energy resolution of  $< 50\text{keV}$ .

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\* URL: <http://www.physics.okayama-u.ac.jp/>

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